Simple formulas for the beam lifetime analysis in an electron storage ring

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Derived are simple and accurate formulas estimating the Touschek and beam-gas scattering lifetimes. These formulas are practical rather than theoretical. They use only the measured total beam lifetime τ and its time derivative $d\tau/dt$ to estimate the two lifetimes. The only condition required is suppression or saturation of the radiative polarization. As a demonstration, the formulas are applied to the Pohang Light Source (PLS) beam lifetime.

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I. INTRODUCTION

The beam lifetime in an electron storage ring is determined mainly by the combination of two types of scatterings, the Touschek scattering between electrons [1], and the elastic and inelastic scatterings between electrons and residual gaseous molecules. We can evaluate each effect by defining the so called Touschek lifetime and beam-gas scattering lifetime separately. There can also be a quantum lifetime which is caused by the fact that the distribution of an electron bunch expands to infinity both transversally and longitudinally, but is cut off by apertures in both directions (vacuum chamber walls and rf power limitation). However, a theoretical derivation shows that the quantum lifetime depends exponentially on the aperture limitations and thus the quantum lifetime is too long to be meaningful for many existing storage rings, which has been supported by experiments and observations (see, e.g., Ref. [2]). Therefore, this paper ignores the quantum lifetime and does not pay attention to the exceptional cases where the quantum lifetime may matter. Then the measured beam lifetime can be analyzed simply as follows:

$$\frac{1}{\tau}=\frac{1}{\tau_t}+\frac{1}{\tau_g}, \tag{1}$$

where τ_t and τ_g denote the Touschek lifetime and beam-gas scattering lifetime, respectively.

The Touschek scattering occurs to electrons in the same bunch. If N is the number of electrons in a bunch, the loss rate due to the Touschek scattering is proportional to N^2, and we may write it as

$$\frac{dN}{dt}=-aN^2, \tag{2}$$

where a is a parameter depending not only on the Möller scattering cross section but also on beam parameters and machine parameters. It is given by (see, e.g., Ref. [3])

$$a=\frac{\sqrt{\pi}cr_e^2}{\gamma^3 V\sigma_{x'}(\Delta p_m/p)^2}C(\epsilon), \tag{3}$$

where r_e is the classical electron radius, $\Delta p_m/p$ is the momentum acceptance, γ is the Lorentz factor, $V=(4\pi)^{3/2}\sigma_x\sigma_y\sigma_l$ is the bunch volume, and the function $C(\epsilon)$ is defined by

$$C(\epsilon)=\epsilon\int_\epsilon^\infty \frac{1}{u^2}\left\{\left(\frac{u}{\epsilon}\right)-\ln\left(\frac{u}{\epsilon}\right)^{1/2}-1\right\}e^{-u}du,$$

$$\epsilon=\left(\frac{\Delta p_m}{\gamma^2\sigma_{x'}mc}\right)^2. \tag{4}$$

The Touschek lifetime τ_t, given by

$$\frac{1}{\tau_t}=-\frac{1}{N}\frac{dN}{dt}=aN, \tag{5}$$

is inversely proportional to N. From Eq. (2), we know that N is a function of time. As a result, $\tau_t(N)$ is also a function of time. Although a has no explicit time dependence, it may have an implicit time dependence through the stored beam current dependence of bunch size factors such as σ_x, σ_y, etc. The bunch size can be increased by intrabeam scattering or instabilities, which decreases as the stored current decays. Hence we will use $\tau(t;a)$ to stress the explicit and implicit time dependences.

On the other hand, the loss rate due to the elastic and inelastic beam-gas scattering is proportional to the product of the total number of electrons and that of residual gaseous molecules. Hence the beam-gas scattering lifetime τ_g is inversely proportional to the vacuum pressure P as in

$$\frac{1}{\tau_g}=bP, \tag{6}$$

where b is a time independent parameter depending not only on machine parameters such as β-function values, vertical aperture of the ring, but also on residual gas information such as gas composition (for details of b, see, e.g., Refs. [3,4]). Note that τ_g has no explicit time dependence. But it may depend on time implicitly through a possible time dependence of P. The pressure P generally depends on the stored current. Since the current decays as time goes on, P can also depend on time. Hence we will use $\tau_g(P)$ to denote the implicit time dependence.

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As shown above, τ_t and τ_g have different characteristics. Knowing them separately and accurately is useful practically. They reflect different parameters of a storage ring and are sensitive monitors of what may be happening on those parameters. If some problems occur, these lifetimes reflect them immediately. However, while it is easy and clear to define the two lifetimes in theory, they are mixed up in real measurements. Lifetimes calculated with known formulas of a and b may not agree with measurements numerically. The purpose of this paper is to present simple and accurate formulas that calculate τ_t and τ_g from the measured values of τ and its time derivative $d\tau/dt$. The derivation is based on the fact that τ_t and τ_g have different time dependences. Here we assume the simple case of an electron beam stored and decaying. We do not assume a situation such as top-up operation.

II. DERIVATION

Consider a beam lifetime measured at time t:

$$\frac{1}{\tau(t;a,P)}=\frac{1}{\tau_t(t;a)}+\frac{1}{\tau_g(P)}. \tag{7}$$

Now we measure τ again at a later time $t'=t+\Delta t$ and find that τ is increased to $\tau+\Delta\tau$. If Δt is big enough and the stored current drops considerably after Δt, the pressure P may drop too, and τ_g may increase noticeably, although the change of P caused by the decaying current is small in established machines. We can choose Δt small enough not to cause the change of P and thus not to increase τ_g. Hence τ_g is effectively time independent, at least within an time interval of the order of Δt and $\Delta\tau$ is caused entirely by the increase of τ_t to $\tau_t+\Delta\tau_t$. At $t'=t+\Delta t$, Eq. (7) becomes

$$\frac{1}{\tau+\Delta\tau}=\frac{1}{\tau_t+\Delta\tau_t}+\frac{1}{\tau_g}. \tag{8}$$

In the first order of approximation,

$$\frac{1}{\tau}-\frac{\Delta\tau}{\tau^2}\simeq\frac{1}{\tau_g}+\frac{1}{\tau_t}-\frac{\Delta\tau_t}{\tau_t^2}. \tag{9}$$

With Eq. (7), we are left with the following relation:

$$\frac{\Delta\tau}{\tau^2}\simeq\frac{\Delta\tau_t}{\tau_t^2}. \tag{10}$$

Now a crucial step is in order. It is the simple relation

$$\Delta\tau_t=\Delta t, \tag{11}$$

which is valid provided a is independent of time. Again, we choose Δt appropriate enough to consider a time independent. Using Eqs. (2) and (5), we compute

$$\frac{d\tau_t}{dt}=\frac{d\tau_t}{dN}\frac{dN}{dt}=\left(-\frac{1}{aN^2}\right)(-aN^2)=1. \tag{12}$$

However, there exists another source of time dependence for a; it is the radiative polarization. The radiative transverse polarization for an initially unpolarized electron beam proceeds according to the formula [5,6]

$$S=S_0(1-e^{-t/T}), \tag{13}$$

where S is the degree of polarization and S_0 is its saturation value. The Touschek lifetime depends on the radiative beam polarization, because the Möller scattering cross section depends on the transverse beam polarization [7]. The Touschek loss rate gets smaller as the degree of beam polarization gets bigger. Hence the Touschek parameter a becomes time dependent through Eq. (13), unless depolarizing devices such as a skew quadrupole are turned on. But, fortunately, the time dependence of S stops after the saturation. The time constant T is given by

$$T\simeq 98\times\frac{R_{bnd}^2R_{avg}}{E^5}[\mathrm{sec}], \tag{14}$$

where E is the beam energy in GeV, and R_{bnd} and R_{avg} are the bending radius and the mean radius of the ring in meters respectively. With the Pohang Light Source (PLS) parameters, $E=2.0$ GeV, $R_{bnd}=6.30$ m, and $R_{avg}=44.65$ m, T is 1.5 h. Equation (14) shows that as the beam energy increases, T decreases rapidly. For higher energy rings, polarization saturates very rapidly and a is effectively time independent except at the very first stage of beam storing.

If we wait an enough time for the saturation or if we turn on depolarizing devices, a is time independent. We use Eq. (11) in Eq. (10) to get

$$\tau_t(t;a)\simeq\frac{\tau(t;a,P)}{\sqrt{\Delta\tau/\Delta t}}, \tag{15}$$

in which τ_t is determined from the measured values of τ, $\Delta\tau$, and Δt. Then τ_g is simply determined by

$$\frac{1}{\tau_g}=\frac{1}{\tau}-\frac{1}{\tau_t}\simeq\frac{1}{\tau}\left(1-\sqrt{\frac{\Delta\tau}{\Delta t}}\right), \tag{16}$$

which is just

$$\tau_g(P)\simeq\frac{\tau(t;a,P)}{1-\sqrt{\Delta\tau/\Delta t}}. \tag{17}$$

The above approximate equalities become exact equalities when $\Delta t\to dt$:

$$\tau_t(t;a)=\frac{\tau(t;a,P)}{\sqrt{d\tau/dt}}, \tag{18}$$

$$\tau_g(P)=\frac{\tau(t;a,P)}{1-\sqrt{d\tau/dt}}. \tag{19}$$

These are exact and practical formulas defining τ_t and τ_g in terms of easily measurable quantities. You need to know only

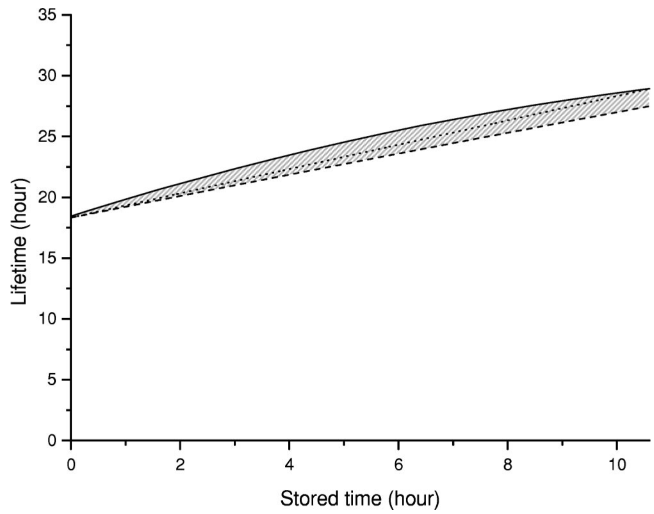

FIG. 1. The solid line represents the measured PLS beam lifetime with skew quadrupoles off, and the dashed line represents the beam lifetime with skew quadrupoles on. Both lines are smoothed ones. The dotted line is a straight line with the time derivative 1 which represents a pure Touschek lifetime. The initial current was 157 mA and the initial lifetime was 18.3 h.

τ and $d\tau/dt$. It is straightforward to show $d\tau_t/dP=0$, as it should be. It is also easy to show $d\tau_g/dt=0$, which means here that τ_g is constant as far as a and P are constant. They are effectively constant in some range of time that depends on machine.

It is practical to use Eqs. (15) and (17) instead of exact relations. For Δt, that belongs to the time range mentioned above, these equations can be applied with a good accuracy. It is easy to determine both τ_t and τ_g from the measured values of τ, $\Delta\tau$, and Δt. This method is very simple and can be very accurate depending upon the accuracy of measured values of τ and $\Delta\tau$. Hence these values should be measured carefully, and possible fluctuations should be averaged out. Generally speaking, this method is more accurate than direct measurements of τ_t or τ_g. To measure one of them directly, we have to eliminate the other effect. For example, one of the methods for measuring τ_g directly is to decrease N to a very small number to increase τ_t to a big value. Then it may be possible to ignore $1/\tau_t$ in Eq. (1) and identify the measured beam lifetime as τ_g. The problem is that practically we cannot increase τ_t indefinitely but have to stop at some value. Often it is not big enough and the estimated τ_g would appear to be smaller than it really is.

III. AN APPLICATION

As a demonstration of this method, the PLS beam lifetime is analyzed here. Figure 1 displays PLS beam lifetimes measured with a stored 2.0-GeV beam. In order to block the radiative polarization, skew quadrupoles were turned on. The strength of skew quadrupoles was chosen appropriately so as not to decrease the dynamic aperture. The dashed line of Fig. 1 denotes the measured beam lifetime with skew quadrupoles turned on. The initial current was 157 mA and the initial lifetime was 18.3 h. The final current was 98 mA. The monitored pressure P showed a slight but non-negligible dependence on the stored current.

To be sure that the radiative polarization is suppressed in the dashed line, it is compared with the dotted line, which is a simple straight line with a slope 1 ($d\tau/dt=1$) representing a pure Touschek lifetime τ_t, and the solid line, which is a measured beam lifetime with skew quadrupoles turned off. The dashed and solid lines have almost the same initial current and initial lifetime. Although τ_t has a slope 1 ($d\tau_t/dt=1$), since the other part τ_g is almost time independent ($d\tau_g/dt\approx 0$) the combined value τ can never have $d\tau/dt$ greater than 1. But in the process of radiative polarization, $d\tau/dt$ has an additional contribution from dS/dt and can be greater than 1. To show this, note that the effect of polarization can be included as follows [8]:

$$\tau_{ts}=\frac{1}{a_s N}, \tag{20}$$

where the subscript s denotes the inclusion of the polarization effect and the new parameter a_s is defined by

$$a_s=a-gS^2, \quad g>0. \tag{21}$$

The parameter g is determined by the polarization dependence of the Möller scattering cross section. The calculation of g is very involved and will not be performed here. $d\tau_{ts}/dt$ is given by

$$\frac{d\tau_{ts}}{dt}=\left(-\frac{1}{a_s N^2}\right)(-a_s N^2)+\frac{d\tau_{ts}}{da_s}\frac{da_s}{dt}=1+\frac{2gS}{a_s}\frac{dS}{dt}, \tag{22}$$

which is always greater than 1. Hence $d\tau/dt$ can be greater than 1 even after including τ_g. The solid line clearly has a slope greater than 1 at the early stage of beam storing. This is evidence of a radiative polarization procedure going on. But the beam lifetime with skew quadrupoles (dashed line) has a $d\tau/dt$ of less than 1 in Fig. 1, which is clear evidence that its polarization has been suppressed completely or to a small value. The skew quadrupoles may also give a slight change to τ_t by modifying the bunch volume, but τ_g is not affected. Hence both the dashed and solid lines have the same τ_g. Once τ_g is determined, τ_t can always be evaluated from Eq. (7) for both the dashed line and the solid line. Note that the hatched area of Fig. 1 denotes approximately the polarization contribution to the beam lifetime.

Equation (17) is now applied to several points of the dashed line (to several values of t) to obtain τ_g at several values of the stored current. In order not to include the effect of dS/dt possibly remaining even after skew quadrupoles were turned on, we used data after $t>2$ h. In each application, the accuracy of the determined τ_g depends entirely upon the measurement accuracy of τ and $\Delta\tau/\Delta t$. Since Δt can be measured very accurately, the accuracy of $\Delta\tau/\Delta t$ is effectively the same as that of τ. Hence splitting τ into τ_g and τ_t is achieved with only the measurement accuracy of τ itself. But it is useful to describe the beam-gas scattering lifetime by its product with the vacuum pressure P, to make $\tau_g P=1/b$ which is supposed to be independent of both the pressure and time. And determining $\tau_g P$ includes additional errors. Besides the measurement error of P, the constancy of $\tau_g P$ may also be slightly broken by the fact that the gas composition changes in general as the pressure P changes by the photoinduced outgasing. In other words, b (and thus $\tau_g P$) may depend on the stored current. This effect is not universal but depends on each machine's status and operation history. For a storage ring of long operation history, the gas composition would change only a little. The measured PLS gas composition is not accurate enough to predict the current dependence of $\tau_g P$ correctly and will not be discussed here. Estimation based on the dashed line shows that values of $\tau_g P$ are not constant but deflect within 10% while the possible current dependence due to the gas composition change is not clear. This is probably because the pressure measurement error was mixed with a gas composition change that was not big enough to show the current dependence within the narrow range of the applied stored current of Fig. 1. The average over the estimated values gives

$$\tau_g \cdot P \approx 160 \quad \text{n Torr h}. \tag{23}$$

On the other hand, a theoretical calculation gives $\tau_g^{cal} \cdot P \approx 145$ n Torr h, which is fairly close to the above value. Also we measured τ_g directly with a very small value of bunch current (0.02 mA/bunch) and obtained $\tau_g^{dir} P \approx 115$ n Torr h [9], which is quite lower than the above value of Eq. (23). The discrepancy between τ_g and τ_g^{dir} can be explained by the reasoning that the Touschek lifetime was not big enough to be ignored in the measurement and it made τ_g^{dir} smaller than τ_g.

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